CALCULATION OF THE KINETICS OF PROPAGATION OF A POWERFUL LIGHT BEAM IN A TRANSPARENT DIELECTRIC WITH IMPURITIES

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A large number of studies have been dedicated to the interaction of powerful optical radiation with transparent dielectrics in the prebreakdown or breakdown regimes. However, the mechanism by which the material is destroyed has not been determined decisively, as indicated by the constant flow of new publications on this theme. Attempts to obtain destruction as the result of electron avalanche [1] give threshold power values orders of magnitude greater than experiment [2]. In connection with this, in recent years the accent has been to deal with the concept of microimpurities of foreign particles or inhomogeneities within the medium having dimensions so small that their presence and concentration is difficult to monitor. As it absorbs optical radiation, the microimpurity (inhomogeneity) is heated and warms the areas of the medium adjacent to itself, which areas then commence to absorb light with significantly more intensity than they did in the initial state [3]. As a result, increase in absorption within the medium commences, terminating in breakdown or destruction of the material around the inhomogeneity. In [4, 5] it was noted that an important role may be played in such a destruction mechanism by thermoelastic stresses in the medium, which factor was not considered in [3]. In [4, 5] it was proposed that the basic effect of thermoelastic stresses reduces to development of microfissures in the medium. However, thermoelastic stresses can lead to yet another effect – narrowing of the forbidden zone of the medium and increase (together with the analogous action of temperature growth) in the coefficient of absorption of the medium. In the present study, the kinetics of interaction of optical radiation with a dielectric medium containing spherical metal particles as an impurity will be calculated, and it will be shown that thermoelastic stresses produce a significant contribution to the increase in light absorption by the medium around a particle.

1. Formulation of the Problem. Thermal Conductivity Equation. We formulate the problem in the following manner. A flux of optical (laser) energy propagates normally from a plane surface into a semiinfinite dielectric layer (of the fused quartz type) with initial intensity I_0. Propagation in the medium commences at time $t=0$ and occurs along the x axis. The dielectric is transparent to the given radiation, but contains an impurity in the form of metallic (platinum) particles which absorb strongly, thus heating up. The particles have the form of spheres with radius R, and their concentration is such that after heating over time intervals considered here the heated and compressed zones of material surrounding the particles still do not overlap each other, but the absorption of particles and heated (compressed) regions is dominant. For the majority of physical parameters in the problem, we will use below values presented in [5] for the case of laser radiation in glasses with platinum inclusions. The present inclusion concentration is significantly greater than in [5]. This is done to demonstrate more graphically how the absorption mechanism considered here can affect the space–time profile of the light flux intensity.

We assume the particle dimensions to be small ($R \approx 5 \cdot 10^{-7}$m), while the volume of medium around the particle in which additional absorption is marked is much larger than R. This allows neglect of the "shadow" beyond the particle both in calculating thermal conductivity and in the elastic part of the problem where for each particle we may assume a spherically symmetric distribution of light absorption by the medium.

With consideration of the above, the thermal conductivity equation describing heating of the medium around a particle due to heat liberation from the particle and also due to absorption by the medium itself can be written in the form

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$$\rho c \frac{\partial T}{\partial t} = \varkappa \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + bI(x, t), \tag{1.1}$$

where ρ is the density of the medium; c, heat capacity; $\varkappa$, thermal conductivity; $b = b_0 e^{-E/kT}$; $2E = E_1$, forbidden zone width (see [3]). The coefficient of light absorption for the medium in the form presented here allows explicit consideration of the effect of heating of the medium and of the stresses developing therein. The conditions for applicability of such an expression for b are given below.

The width of the forbidden zone is a function of pressure. Relying on data available in the literature (see, e.g., [6]) on the character of the dependence of gap width on pressure, for the time interval under consideration here we may take a linear dependence of E on stress, $E = E_0 + \beta\sigma_{rr}$, where σ_{rr} is the radial component of the elastic stress tensor [7], developing upon heating, and $\beta \approx 0.001\ kT_0/p_0$ (where p_0, T_0 are the initial pressure and temperature).

The boundary condition for Eq. (1.1) has the form

$$\frac{4}{3}\pi R^3 \rho_1 c_1 \left.\frac{\partial T}{\partial t}\right|_{r=R} = \pi R^2 \alpha_0 I + \varkappa \left.\frac{\partial T}{\partial r}\right|_{r=R} 4\pi R^2, \tag{1.2}$$

where ρ_1 is the density of the particle; c_1, its heat capacity; and α_0, absorption coefficient of the metallic surface; Eq. (1.2) relates the energy absorbed by the particle to particle heating and heat liberation into the medium. In the calculations the values $\rho = 3 \cdot 10^3$ kg/m^3, $c = 1.3 \cdot 10^3$ J/kg·deg K, $\varkappa = 1.3$ W/m·deg K, $\rho_1 = 2 \cdot 10^4$ kg/m^3, $c_1 = 1.3 \cdot 10^2$ J/kg·deg K, $I_0 = 2 \cdot 10^{11}$ W/m^2 were used, taken from [5]. In addition, $\alpha_0 = 0.3$, $T_0 = 300°$K, $p_0 = 10^5$ N/m^2. For E_0 the value $E_0 = 60\ kT_0$ was used, realistic for the given type of medium.

2. Thermoelastic Stresses. The stresses that develop in the medium and particle will be sought commencing from the assumption of brevity of the stress field establishment time in comparison to the characteristic times of temperature field variation. The rate of temperature field variation will be determined by the temporal behavior of radiation intensity, since the thermal conductivity mechanism is slower. Assuming, as in [5], a pulse duration of $3 \cdot 10^{-8}$ sec, we then characterize the temperature rate of change by times on the order of $\tau_1 \sim 10^{-9}$ sec. Elastic stresses will be transferred through the medium at the speed of sound, $3 \cdot 10^3$ m/sec. Taking the ratio of the particle size, $5 \cdot 10^{-7}$ m, to the speed of sound, we find the characteristic time for variation of the stress field, $\tau_2 \sim 10^{-10}$ sec. Thus, with satisfactory qualitative accuracy, $\tau_1 \gg \tau_2$. This allows determination of the thermoelastic stress distribution from a quasistationary temperature distribution. In such a formulation, the thermoelasticity problem has an analytic solution [7]. In fact, the equation for displacements of points of matter u(r) has the form [7]

$$\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial (r^2 u)}{\partial r}\right) = \alpha \frac{1+\sigma}{3(1-\sigma)}\frac{\partial T}{\partial r}, \tag{2.1}$$

where α is the coefficient of volume expansion and σ is the Poisson coefficient. With a known T(r), solution of Eq. (2.1) presents no problem. On the other hand, we may use u(r) to express the elastic stress tensor

$$\sigma_{rr} = \frac{E}{(1+\sigma)(1-2\sigma)}\left[(1-\sigma)\frac{\partial u}{\partial r} + \frac{2\sigma u}{r} - \frac{\alpha}{3}(1+\sigma)(T - T_0)\right]. \tag{2.2}$$

It should be noted that, with significant heating, it is necessary to consider the temperature dependence of E, σ, α. In the variant presented below, the temperature change is relatively small, and consideration of the dependence of E, σ, α on T would lead only to small corrections; thus it was not done. Using the assumption of infinitely high thermal conductivity of the particle material, solving Eq. (2.1) for the particle and the medium, matching the two solutions (the matching condition being equality of displacements and stresses on the boundary, with zero values of the same quantities at infinity), then using Eq. (2.2), we find

$$p_2 - 2\left[\frac{\alpha_1 (T(R) - T_0)}{3} - \frac{\alpha_2}{r_{10}^3}\int_1^{r_{10}} (T(r_1) - T_0)\, r_1^2 dr_1\right]\left[\frac{2(1-2\sigma_1)}{E_1} + \frac{(1+\sigma_2)}{E_2}\right]^{-1}, \tag{2.3}$$

$$u_2(R) = u_1(R) = \left[\frac{\alpha_1 (T(R) - T_0)}{3} - \frac{p_2(1-2\sigma_1)}{E_1}\right],$$

$$\sigma_{rr}^1 = \frac{E_2}{(1+\sigma_2)(1-2\sigma_2)}\left[(1-\sigma_2)\left(-\frac{2u_2(R)}{r_1^3} + 2u_2(R)\frac{(1-2\sigma_2)}{(1-\sigma_2)} - \right.\right.$$

$$- p_2 \frac{(1+\sigma_2)(1-2\sigma_2)}{(1-\sigma_2)E_2} - \frac{2}{r_1^3}\int_1^{r_1}\left[2u_2(R)\frac{(1-2\sigma_2)}{(1-\sigma_2)} + \alpha_2 \frac{(1+\sigma_2)}{3(1-\sigma_2)}(T(r_1) - T_0) - \right.$$

$$- p_2 \frac{(1+\sigma_2)(1-2\sigma_2)}{E_2(1-\sigma_2)} \Big] r_1^2 dr_1 \Big) + \frac{2\sigma_2}{r_1} \left(\frac{u_2(R)}{r_1^2} + \frac{1}{r_1^2} \int_1^{r_1} \Big[2u_2(R) \times \right.$$

$$\left. \times \frac{(1-2\sigma_2)}{(1-\sigma_2)} + \alpha_2 \frac{(1+\sigma_2)}{3(1-\sigma_2)} (T(r) - T_0) - p_2 \frac{(1+\sigma_2)(1-2\sigma_2)}{(1-\sigma_2)E_2} \Big] r_1^2 dr_1 \right) \Big],$$

where $r_1 = r/R$; $\sigma_{rr}^1 = \sigma_{rr}/p_0$, and the dimensionless quantities $E_{1,2} = E/p_0$, $u_{1,2} = u/R$ are introduced, with index 1 referring to the particle and index 2 to the medium, while r_{10} is chosen from the consideration that the volume around all particles should correspond to the total volume of material. Since the quantities T and σ_{rr} fall off quite rapidly and the major contribution to absorption is produced by the zone of material about the particles with a radius on the order of magnitude of 5R (as shown by calculations), the value of r_{10} may be chosen close to 5R. Equation (2.3) completely defines the thermoelastic stress field in the case where the behavior of T(r) is known. In the calculations the values $E_1 = 1.47 \cdot 10^6$, $E_2 = 7 \cdot 10^5$, $\alpha_1 = \alpha_2 = 0.008/T_0$, $\sigma_1 = 0.39$, $\sigma_2 = 0.22$ were used (see [5]).

3. Kinetic Equation for Light Propagation. In writing the kinetic equation for light propagation, we will assume that the particles have an identical radius and are distributed uniformly throughout the material, while the light flux has a relatively small cross section so that the light scattered by a particle leaves the zone under consideration and is lost. In cases realizable in practice, it is necessary to consider particle scattering over size. This is done by introducing a corresponding distribution function and averaging all calculated quantities over this distribution. It was noted in [3] that with such an averaging, the major role is played by particles with large R, since it is that they lead to the greatest heating of the medium. Thus, our case corresponds to a quite narrow particle distribution over size with an abrupt slope on the large R side. Such a distribution can be obtained in practice when it is possible to "filter off" all particles with a size exceeding some specified value.

Absorption by the medium is determined by the coefficient $b = b_0 e^{-E/kT}$. The expression for b is written in the equilibrium form. Such notation is valid for systems in which the processes of electron-hole relaxation occur over times less than 10^{-8} sec (the time interval considered here). It can be assumed that population relaxation in the regions of the medium considered here will be accelerated by the presence of the metallic particle surface, since the diffusion length of electrons and holes in a medium of the given type can markedly exceed the zone size $5R \approx 3 \cdot 10^{-6}$ m. This fact should make the above condition less rigid. To find b_0 the value $b_0 e^{-E_0/kT_0} = 0.25\ m^{-1}$ [5] was used. With consideration of absorption by the medium and the particle surface, we find

$$\frac{\partial I}{\partial t} + c_0 \frac{\partial I}{\partial x} = - N\pi R^2 c_0 \left(1 + \int_1^{r_{10}} 4b_0 R\, e^{-E/kT} r_1^2 dr_1 \right) I \qquad (3.1)$$

with boundary condition $I(t, x=0) = I_0$ and initial conditions $I(t=0, x) = 0$. For the particle concentration the value $N = 10^9\ m^{-3}$ was used. In Eq. (3.1) in the parentheses on the right we have 1 instead of α_0, which considers total loss of light scattered by particles.

Equation (3.1) is not completely correct. Integration is performed over spheres of radius r_{10}, which may not intersect the entire volume of the material. Therefore, the following calculation method was used. The value of r_{10} was chosen such that within a zone of radius r_{10} the major portions of the changes T(r) and $\sigma_{rr}(r)$ produced by particle heating were confined. In the calculations the value $r_{10} = 7R$ was chosen, while the value of $\sigma_{rr}(r)$ fell off in this interval by a factor of 10^3 times, and the temperature practically reached T_0. Since heating of the portion of the medium not considered by r_{10} is negligibly small, for that region we take $b = b_0 e^{-E_0/kT_0}$. Multiplying this quantity by the ratio of the difference between the initial volume and the volume of the spheres of radius r_{10} to the initial volume, we find the effective absorption coefficient of the medium, which will be introduced into Eq. (3.1), $-b_0 e^{-E_0/kT_0}\left(1 - \frac{4\pi}{3} r_{10}^3 N\right)$.

Equation (3.1) then transforms to

$$\frac{\partial I}{\partial t} + c_0 \frac{\partial I}{\partial x} = - N\pi R^2 c_0 \left(1 + \int_1^{r_{10}} 4b_0 R\, e^{-E/kT} r_1^2 dr_1 \right) I - c_0 b\, e^{-E_0/kT_0} \left(1 - \frac{4\pi}{3} r_{10}^3 N \right) I, \qquad (3.2)$$

which together with Eqs. (2.3), (1.1), (1.2) produces a closed system which was solved numerically.

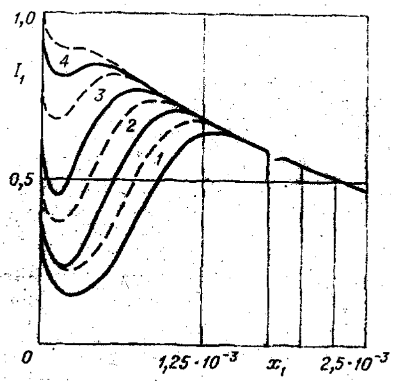

Fig. 1

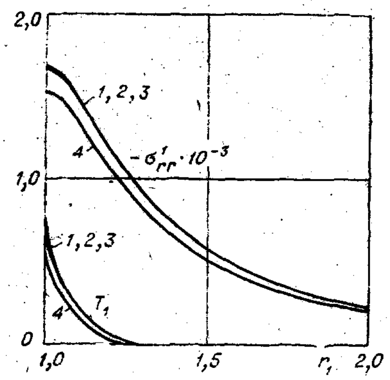

Fig. 2

4. Computation Results. Evaluation. The results of a numerical solution of Eqs. (3,2), (2.3), (1.1), (1.2) are shown in Figs. 1, 2 (1–10^{-8}, 2) $9\cdot10^{-9}$, 3) $8\cdot/10^{-9}$,4) $7\cdot10^{-9}$ sec). Figure 1 shows the dependence of intensity, normalized to the initial value, $I_1=I/I_0$, on the dimensionless coordinate $x_1=x/c_0\tau$, where $\tau = 1/(\pi N R^2 c_0) \approx 4\cdot10^{-6}$ c. The solid lines correspond to functions obtained with consideration of the increase in absorption by the medium due to heating, as well as to the stresses which develop. The dashed lines correspond to consideration of medium heating only. The calculation was performed up to a time value $t=10^{-8}$ sec. It is evident that over such a time strong supplementary absorption by areas around particles can develop, leading to nonmonotonic behavior of intensity and a strong drop in the latter over a period of time of $\sim10^{-8}$ sec after arrival of the light at a given point. The intensity falloff obtained corresponds to the "shadow wave" of [8] and to that previously obtained in the simpler qualitative model of [9].

Thus, the forward segment of the light flux experiences basically absorption by an unheated medium and only a small fraction of the absorption takes place on particles. But, this small fraction eventually leads to heating of the particle, and then of the surrounding medium, producing significant thermoelastic stresses and growth in absorption by the medium itself. A strongly absorbing zone develops around the particle, processes in the medium lose their linearity, and an intensity falloff is formed. Absorption begins to be determined by heated and compressed zones of the medium around the particle. Comparison of the solid and dashed lines of Fig. 1 permits clarification of the effect of thermoelastic stresses on absorption growth in comparison to the analogous action of temperature. The contribution of thermoelastic stresses proves to be significant.

Figure 2 is an aid in understanding why this is true. Figure 2 shows the behavior of the quantities $T_1=T/T_0-1$ and σ^1_{rr} as functions of $r_1=r/R$ at the point $x_1=2.5\cdot10^{-4}$. Most remarkable is the fact that the temperature drop occurs more rapidly than the thermoelastic stress falloff. It is evident from Fig. 2 that the compression zone around the particle proves to be much larger than the heated zone. It is because of this that despite the functionally weaker dependence of medium absorption on stress, as compared to the temperature dependence, the region of contribution from thermoelastic stress is significantly larger than the region where temperature factors act. This is reflected in the increased role of thermoelasticity and the necessity of its consideration in determining the conditions for destruction of the medium.

LITERATURE CITED

1. P. M. Mednis and V. M. Fain, "Excitation of avalanche ionization in transparent dielectrics by a strong ac electromagnetic field," Zh. Éksp. Teor. Fiz., 62, No. 2 (1972).
2. The Action of Laser Radiation [Russian translation], Mir, Moscow (1968),
3. I. V. Aleshin, S. I. Anisimov, A. M. Bonch-Burevich, Ya. A. Imas, and E. L. Komolov, "Optical breakdown of transparent media containing microinhomogeneities," Zh. Éksp. Teor. Fiz., 70, No. 4.
4. R. W. Hopper and D. R. Uhlmann, "Mechanism of inclusion damage in laser glasses," J. Appl. Phys., 41, No. 10 (1970).
5. H. S. Bennett, "Absorption centres in laser materials," J. Appl. Phys., 42, No. 2 (1971).
6. B. Welber, M. Cardona, C. K. Kim, and S. Rodriquez, "Dependence of the direct gap of GaAs on hydrostatic pressure," Phys. Rev. B, Solid State, 12, No. 12 (1975).
7. L. D. Landau and E. M. Lifshits, Theory of Elasticity [in Russian], Nauka, Moscow (1965).
8. A. M. Bonch-Burevich, Ya. A. Imas, M. N. Libenson, V. S. Salyadinov, G. D. Shandybina, and E. B. Yakovlev, "Clarification and shadow waves in silicon induced by multimode laser radiation," Zh. Tekh. Fiz., 47, No. 3 (1977).
9. Yu. I. Lysnikov and I. A. Shamsutdinov, "Anomalous behavior of light flux intensity in a medium with two types of absorption," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1976).

INDUCTANCE OF A SINGLE-COIL MAGNETIC COURSE GENERATOR WITH A VARIABLE GENERATING COIL

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1. Several types of explosive magnetic generators are known at the present time [1]. Of these (according to published experimental data) the most effective are the coaxial [2], plane-parallel "busbar–busbar" type [3, 4], and the "bellows" type [5]. A high magnetic-flux storage factor of $\eta \sim 80\%$ is achieved in these generators due to the linear increase in the transverse cross section of the current-carrying conductors (busbars) in a region adjacent to the inductive load.

The initial inductance L_0 of plane-parallel explosive magnetic generators depends on the geometrical dimensions of the current-carrying conductors and, in particular, is proportional to their length. Hence, in such generators a high current gain ($k_T = (L_0/L_H)\eta$, L_H is the load inductance) is achieved due to the long length of the conductors, which leads to a high generator operating time of ~ 100 μsec.

The dimensions and operating time of plane-parallel explosive magnetic generators can be reduced severalfold by placing the busbars around a cylindrical conducting tube with the charge of explosive material along the axis. In [6] the operating time of the generator was reduced to 20 μsec in this way. The current-carrying conductors in this generator (a single-coil generator) had a constant cross section, which reduced η at the end of the generator operation, when the magnetic field strength reached a value higher than the critical value H_* for the conductor material (e.g., for copper conductors $H_* \sim 1$ MOe [1]).

The efficiency of a single-coil generator can be increased by shaping its current contour by a parabolic increase in the generating coil from the current terminal to the inductive load [7]. This form of variation of the generating winding is similar in form to the increase in the generator current and limits the increase in the linear current density $j_+(t) = \eta(t)\Phi_0/L(t)z_+(t)$ (the magnetic field strength $H_+(t) = 0.4\pi j_+(t)$) along the line of dynamic contact of the envelope with the coil $z_+(t)$ while the generator is operating. By choosing the geometrical dimensions and the value of the initial magnetic flux of the generator Φ_0, one can ensure the maximum permissible current mode of the conductors (with respect to the field value $H_+(t) \simeq 1$ MOe), which is not exceeded during the operation of the generator at the stage of the electrical disruption of the skin surface of the conductors. This mode of operation is the most convenient for producing miniature explosive magnetic generators with specified electromagnetic parameters, viz., energy and power.

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